

Primordial magnetic field from non-inflationary cosmic expansion in Hořava-Lifshitz gravity

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The origin of large-scale magnetic field in the universe is one of the greatest mysteries in modern cosmology. We present a new mechanism for generation of large-scale magnetic field, based on the power-counting renormalizable theory of gravitation recently proposed by Hořava. Contrary to the usual case in general relativity, the $U(1)$ gauge symmetry of a Maxwell action in this theory permits terms breaking conformal invariance in the ultraviolet. Moreover, for high frequency modes, the anisotropic scaling intrinsic to the theory inevitably makes the sound horizon far outside the Hubble horizon. Consequently, non-inflationary cosmic expansion in the early universe naturally generates super-horizon quantum fluctuations of the magnetic field. Specializing our consideration to the case with the dynamical critical exponent $z = 3$, we show an explicit set of parameters for which (i) the amplitude of generated magnetic field is large enough as a seed for the dynamo mechanism; (ii) backreaction to the cosmic expansion is small enough; and (iii) the high-energy dispersion relation is consistent with the most recent observational limits from MAGIC and FERMI.

I. INTRODUCTION

Hořava recently proposed a class of power-counting renormalizable theories of gravity [1]. The power-counting (super-)renormalizability stems from the Lifshitz-type anisotropic scaling

$$t \rightarrow b^z t, \quad \vec{x} \rightarrow b \vec{x}, \quad (1)$$

with the dynamical critical exponent $z = 3$ (or $z > 3$). Because of this scaling, the theory is often called Hořava-Lifshitz gravity. Although renormalizability of matter action, e.g. the standard model action, does not require the anisotropic scaling, quantum corrections should generate terms leading to the anisotropic scaling with a common z for all physical degrees of freedom in the ultraviolet (UV). For these reasons, in the present paper we shall seriously consider the anisotropic scaling with $z \geq 3$ for matter degrees of freedom, especially for photon.

Note that the value of z (≥ 3) in the UV is a part of the definition of a theory, provided that the theory is renormalizable. Once z in the UV is fixed then terms leading to higher z would not be generated by quantum corrections. In this paper, for simplicity, we shall restrict our consideration to the simplest case where z in the UV is 3. However, in principle the mechanism presented in this paper works for any values of z in the UV. (See the last paragraph of Sec. VI for a comment on the case with general $z > 3$.)

Cosmology based on this theory has been investigated by many authors, and number of interesting implications have been pointed out [2, 3, 4, 5, 6, 7]. Among them, the one particularly relevant to the present paper is that the anisotropic scaling of a physical degree of freedom leads to a new mechanism for generation of super-horizon quantum fluctuations without inflation [4].

Needless to say, the driving force behind recent enthusiasm for cosmology based on Hořava-Lifshitz gravity is the fact that this theory is a new candidate for

quantum gravity. At this moment, it is not yet clear if some version of Hořava-Lifshitz gravity makes sense at the quantum level and can be applied to the real world. Indeed, the version without the projectability condition is already known to be problematic [8]. On the other hand, problems pointed out in the literature are absent if the projectability condition is maintained and if the detailed balance condition is abandoned [5]. Therefore, Hořava-Lifshitz gravity with the projectability condition without the detailed balance condition has a potential to be theoretically consistent and phenomenologically viable. While there still remain many issues to be addressed in the future, we may regard Hořava-Lifshitz theory as a candidate for the UV completion of general relativity. For this reason, it is interesting and important to investigate cosmological implications of the theory.

In this paper we shall focus on the origin of large-scale magnetic field in the universe. The observed magnitude of the magnetic field at scales of galaxies and clusters is about $1\mu\text{Gauss}$. At larger scales, on the other hand, there is only an upper limit ($< 10^{-9}\text{Gauss}$) from e.g. observation of the cosmic microwave background [9]. Galactic magnetic field can be amplified by the so-called dynamo mechanism [10], but a seed magnetic field must be provided by other mechanisms in an earlier epoch since the dynamo mechanism does not generate magnetic field from nothing. As for scales of clusters, efficient amplification mechanism from tiny primordial magnetic fields to the observed amplitudes has not been established. In this sense there is no consensus about how observed magnetic fields at cluster scales could be related to primordial ones. However, it has been suggested that, once galactic magnetic field is amplified by the dynamo mechanism, those amplified magnetic fields can spread over cluster scales through galactic outflows (winds/active galactic nuclei ejecta) [11] or some plasma instabilities [12]. For these reasons, in the present paper, we shall restrict our consideration to the origin of primordial magnetic seed

fields responsible for galactic magnetic fields.

Various generation mechanisms of the seed magnetic field have been proposed so far. Among them, scenarios based on cosmic inflation are popular [13, 14]. (See, however, Refs. [10, 15, 16, 17, 18] for other scenarios.) This is largely because inflation provides a natural framework in which physical wavelengths of quantum fluctuations are stretched by rapid expansion to scales beyond the Hubble horizon. However, this (quasi-)standard paradigm based on inflation was recently challenged on the basis of strong backreaction and strong coupling [19]. If we simply demand that the generated magnetic field does not backreacts to the dynamics of inflation significantly and that the coupling constant in the theory is not extremely large then the amplitude of the primordial seeds cannot exceed 10^{-32}G in Mpc scales. Since a large coupling constant makes any perturbative calculations untrustable, it is necessary to take the strong backreaction into account or/and to analyze the strongly-coupled quantum dynamics non-perturbatively. In some model of inflationary magnetogenesis, the effect of backreaction was investigated and it was claimed that the magnetic field generated by inflation significantly backreacts to the cosmic expansion in such a way that generation of magnetic field is, after all, highly suppressed. At present it is not clear whether the strong backreaction or/and the strong coupling really spoils other models of inflationary magnetogenesis or not. While it is certainly worthwhile investigating this issue in more details, it is also plausible to look for alternative mechanisms.

It is well known that conformal invariance of the standard Maxwell action prevents cosmic expansion (including inflation) from acting as a generation mechanism of magnetic field [20]: the Maxwell field in the flat Friedmann-Robertson-Walker universe does not feel cosmic expansion and behaves as if it were in flat spacetime. Therefore, any generation mechanisms need to include, one way or another, effects breaking conformal invariance.

Interestingly enough, in Hořava-Lifshitz gravity, the $U(1)$ gauge symmetry of a Maxwell action permits terms breaking conformal invariance. Actually, among them, most important in the UV are those associated with the anisotropic scaling (1). Therefore, breaking of conformal invariance is not only possible but also inevitable in the UV regime of Hořava-Lifshitz gravity. Moreover, as already stated, super-horizon quantum fluctuations can be generated without inflation [4]. The essential reason is that the sound horizon for high frequency modes is far outside the Hubble horizon if a physical degree of freedom exhibits the anisotropic scaling.

The rest of this paper is organized as follows. In Sec. II we describe the action for an electromagnetic field in Hořava-Lifshitz theory. In Sec. III we describe our mechanism for generation of magnetic fields and present the power spectrum of the magnetic field. The results obtained by qualitative scaling arguments there will be confirmed by explicit calculations in Appendix. In Sec. IV we

shall investigate the backreaction problem raised in Ref. [19] and confirm that the backreaction is small enough for a wide range of parameters. Then we shall estimate the order of magnitude of the generated magnetic field in Sec. V. We shall show an explicit set of parameters for which (i) the amplitude of generated magnetic field is large enough as a seed for the dynamo mechanism; (ii) backreaction to the cosmic expansion is small enough; and (iii) the high-energy dispersion relation is consistent with the most recent observational limits from MAGIC [21] and FERMI [22]. Finally, Sec. VI is devoted to a summary of this paper.

II. ELECTROMAGNETIC FIELD IN HOŘAVA-LIFSHITZ THEORY

In Hořava-Lifshitz theory, gravity is described by three basic quantities: the lapse function $N(t)$, the shift vector $N^i(t, \vec{x})$ and the three-dimensional spatial metric $g_{ij}(t, \vec{x})$. We can combine these three to form a 4-dimensional metric of the ADM form:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (2)$$

The fundamental symmetry of the theory is invariance under the foliation-preserving diffeomorphism:

$$t \rightarrow t'(t), \quad \vec{x} \rightarrow \vec{x}'(t, \vec{x}). \quad (3)$$

This symmetry, combined with the value of the dynamical critical exponent z (≥ 3) in the UV, completely determines the structure of the gravitational action [1, 23].

In this paper we investigate the dynamics of the electromagnetic field, i.e. a $U(1)$ gauge field, in Hořava-Lifshitz theory. The basic quantities are the scalar potential $A_0(t, \vec{x})$ and the vector potential $A_i(t, \vec{x})$. The (generalized) Maxwell action must respect the $U(1)$ gauge symmetry as well as the foliation-preserving diffeomorphism invariance. As in the gravity sector, these symmetries, combined with the value of the dynamical critical exponent z (≥ 3) in the UV, completely determine the structure of the action. The (generalized) Maxwell action is, thus,

$$S = \frac{1}{4} \int N \sqrt{g} dt d^3 \vec{x} \left[\frac{2}{N^2} g^{ij} (F_{0i} - N^k F_{ki}) \times (F_{0j} - N^l F_{lj}) - G[B_i] \right], \quad (4)$$

where $F_{0i} = \partial_0 A_i - \partial_i A_0$, $F_{ij} = \partial_i A_j - \partial_j A_i$, and $G[B_i]$ is a function of the magnetic field B_i and its spatial derivatives. The magnetic field is defined, as usual, by

$$B_i = \frac{1}{2} \epsilon_{ijk} g^{jl} g^{km} F_{lm}, \quad (5)$$

where ϵ_{ijk} is the totally anti-symmetric tensor with $\epsilon_{123} = \sqrt{g}$. Restricting our consideration to the case

where A_i is a free field, $G[B_i]$ in general has the form

$$G[B_i] = a_1 B_i B^i + a_2 g^{ik} g^{jl} \nabla_i B_j \nabla_k B_l + a_3 g^{il} g^{jm} g^{kn} \nabla_i \nabla_j B_k \nabla_l \nabla_m B_n + \dots, \quad (6)$$

where a_1 , a_2 and a_3 are constants and ∇_i is the spatial covariant derivative compatible with g_{ij} . The highest derivative term in $G[B_i]$ is the square of the $(z-1)$ -th derivative of the magnetic field. The (generalized) Maxwell action (4) is a special case of the vector field action considered in Ref. [3].

It is easy to see that the scaling dimension of A_0 and A_i are $(z+1)/2$ and $(3-z)/2$, respectively. This will be important for the estimate of the power-spectrum of the magnetic field in the next section.

One of most important properties of Hořava-Lifshitz theory is that in the UV the theory exhibits the anisotropic scaling with the dynamical critical exponent $z = 3$ ¹. As already stated in introduction, this property should be shared with matter fields such as the electromagnetic field. Therefore, the function $G[B_i]$ in the UV should be dominated by the $z = 3$ term

$$G[B_i] \ni \frac{1}{M^4} g^{il} g^{jm} g^{kn} \nabla_i \nabla_j B_k \nabla_l \nabla_m B_n, \quad (7)$$

where M is a mass scale defined by $a_3 = 1/M^4$. From the stability of the system in the UV, the sign of this term is required to be positive. For lower energy scales, relevant deformations, i.e. terms with less number of spatial derivatives, become important.

Before closing this section, let us briefly mention observational bounds on M . The electromagnetic field in our model has dispersion relation²

$$\omega^2 \simeq \frac{k_{phys}^6}{M^4} + \kappa \frac{k_{phys}^4}{M^2} + k_{phys}^2, \quad (8)$$

where k_{phys} is the physical wavenumber and κ is defined by $a_2 = \kappa/M^2$. This leads to the energy-dependent photon velocity

$$v = \frac{d\omega}{dk_{phys}} = \frac{k_{phys}}{\omega} \left(1 + 2\kappa \frac{k_{phys}^2}{M^2} + 3 \frac{k_{phys}^4}{M^4} \right) \simeq 1 + \frac{3}{2} \kappa \frac{k_{phys}^2}{M^2} + \left(\frac{5}{2} - \frac{5}{8} \kappa^2 \right) \frac{k_{phys}^4}{M^4} + O\left(\frac{k_{phys}^6}{M^6} \right) \quad (9)$$

¹ Theories with the dynamical critical exponent larger than 3 are power-counting super-renormalizable and, thus, worthwhile considering. In this paper, for simplicity, we consider the case with $z = 3$ only.

² Rigorously speaking, each coefficient on the right hand side is subject to logarithmic running under renormalization group flow. How they actually run has not yet been investigated in full details. (See, however, Ref. [24] for analysis in a simplified setup.) Among them, the running of the coefficient of k_{phys}^2 is required to be small. In the present paper, we suppose that this is already achieved by tuning various coupling constants. As for other coefficients, argument in the present paper does not depend on their precise values and we can treat them as constant unless they change by many orders of magnitude.

For $\kappa = O(1)$, the leading correction to the photon velocity comes from the term proportional to k_{phys}^2/M^2 . In this case the MAGIC Collaboration [21] and the Fermi GBM/LAT Collaborations [22] give similar lower bound on M :

$$M > 10^{11} \text{ GeV}. \quad (10)$$

If $|\kappa| \ll 1$ then the leading correction comes from the term proportional to k_{phys}^4/M^4 and the lower bound on M is weaker.

III. GENERATION OF SUPER-HORIZON SCALE MAGNETIC FIELD WITHOUT INFLATION

Let us consider the electromagnetic field described by the (generalized) Maxwell action (4) in the flat Friedmann-Robertson-Walker (FRW) spacetime. The metric is given by

$$\begin{aligned} ds^2 &= -dt^2 + g_{ij} dx^i dx^j \\ &= -dt^2 + a^2 \delta_{ij} dx^i dx^j \\ &= a^2 [-d\eta^2 + \delta_{ij} dx^i dx^j], \end{aligned} \quad (11)$$

where a is the scale factor and η is the conformal time. The Latin indices run over spatial index ($i = 1, 2, 3$).

Let us first look at the freeze-out condition by assuming the power-law expansion for the background universe [4]. As seen easily from the action, the dispersion relation for the vector potential will be significantly modified as

$$\omega \sim k^z / a^z M^{z-1}, \quad (12)$$

where ω is the physical frequency and k is the comoving wavenumber. In the UV the dynamical critical exponent z is 3 but we shall leave it as a free parameter for a while until we need to specify it. Hereafter, for simplicity we adopt the unit with $M = 1$. The fluctuation is expected to oscillate (freeze out) if $\omega \gg H$ ($\omega \ll H$). Thus if

$$\partial_t (a^{2z} H^2) > 0. \quad (13)$$

is satisfied, vector fields first oscillate and then freeze out afterwards. For the power-law expansion, $a \propto t^p$, the condition of Eq. (13) becomes $p > 1/z$. Throughout this paper we consider cases satisfying the above condition since in this case super-horizon quantum fluctuations of vector fields can be generated without inflation.

Now we compute the power spectrum of the magnetic field in a qualitative way (See Appendix for quantitative analysis). It is easy to guess the scale dependence of it by using the scaling dimension of the fields. The vector potential A_i has the kinetic term of the form

$$\frac{1}{2} \int d\eta d^3 \vec{x} (\partial_\eta A_i)^2. \quad (14)$$

Here remember that a canonically normalized scalar field has the kinetic term of the form $\frac{1}{2} \int d\eta d^3 \vec{x} a^2 (\partial_\eta \phi)^2$ [2, 4]. Thus, the behavior of $\tilde{A}_i \equiv A_i/a$ should be similar to a canonically normalized scalar field. For simplicity, we consider a power-law expansion

$$a \sim \alpha \eta^q \propto t^p, \quad (15)$$

where we see from the definition of the conformal time that $t \sim \alpha \eta^{1+q}$, $p = q/(1+q)$ and $q = p/(1-p)$. For this background, the ratio of H to ω becomes

$$\frac{H}{\omega} \sim \frac{H a^z}{k^z} \sim \alpha^{z-1} k^{-z} \eta^{qz-q-1}. \quad (16)$$

When $H \sim \omega$, all relevant time scales agree. Thus, at the *sound-horizon* crossing $H \sim \omega$, the power-spectrum of the “canonically normalized” field \tilde{A}_i should follow from the scaling dimension of \tilde{A}_i as

$$\begin{aligned} \mathcal{P}_{\tilde{A}_i}|_{H \sim \omega} &\sim H^{(3-z)/z} \Big|_{H \sim \omega} \\ &\sim \alpha^{-(3-z)/z} \eta^{-(q+1)(3-z)/z} \Big|_{H \sim \omega} \\ &\sim \left[\alpha k^{-(q+1)} \right]^{(3-z)/(qz-q-1)}, \end{aligned} \quad (17)$$

where we used $k^z \sim \alpha^{z-1} \eta^{qz-q-1}$ which holds for $H \sim \omega$.

From the kinetic term of Eq.(14), it is easy to see that the super-horizon growing mode behaves as

$$A_i|_{H \gg \omega} \propto \eta. \quad (18)$$

Thus, at the super-horizon scale, the time evolution of $\mathcal{P}_{\tilde{A}_i}$ is given by

$$\mathcal{P}_{\tilde{A}_i}|_{H \gg \omega} \propto \frac{\eta^2}{a^2} \propto \eta^{2(1-q)} \propto \left(\frac{H}{\omega} \right)^{2(1-q)/(qz-q-1)}, \quad (19)$$

where we used Eq. (16) in the last. For the moment, we did not take care of the wavenumber dependence. Recovering the wavenumber dependence, then, we obtain

$$\begin{aligned} \mathcal{P}_{\tilde{A}_i}|_{H \gg \omega} &\sim \mathcal{P}_{\tilde{A}_i}|_{H \sim \omega} \times \left(\frac{H}{\omega} \right)^{2(1-q)/(qz-q-1)} \\ &\sim \alpha^m k^{n-2} \frac{\eta^2}{a^2}, \end{aligned} \quad (20)$$

where

$$m := \frac{z-1}{qz-q-1}, \quad n := 5 - \frac{z}{qz-q-1}. \quad (21)$$

Thus, we compute $\mathcal{P}_{A_i}|_{H \gg \omega}$ as

$$\mathcal{P}_{A_i}|_{H \gg \omega} \sim \alpha^m k^{n-2} \eta^2, \quad (22)$$

and the power spectrum of the magnetic field is

$$\mathcal{P}_B \sim \frac{k^2}{a^4} \mathcal{P}_{A_i} \sim \alpha^m k^n \frac{\eta^2}{a^4}. \quad (23)$$

So far, we have been working in the unit with $M = 1$. Noting that \mathcal{P}_B has mass dimension four, we can easily recover M as

$$\mathcal{P}_B \sim (\alpha M)^m k^n \frac{\eta^2}{a^4}. \quad (24)$$

(See Appendix for explicit confirmation of this result.)

The correlation length of the generated magnetic fields is roughly the sound-horizon size. Since the anisotropic scaling in the UV regime makes the sound horizon far outside the Hubble horizon, the magnetic field on super-horizon scales can be generated. Hence the correlation length naturally becomes the cosmological scale.

We find that time evolution of the power spectrum of the super-horizon magnetic field is proportional to η^2/a^4 . Here if a is the de Sitter expansion $a \propto 1/\eta$, then we obtain $\mathcal{P}_B \propto a^{-6}$. Thus, the generated magnetic field rapidly decays. In this sense inflationary universe is not good for the generation of the magnetic field in our model. Therefore we will not consider inflationary phases.

IV. ABSENCE OF BACKREACTION PROBLEM

As recently pointed out in Ref. [19], we have to check if the generated magnetic fields affect the background universe. Let us suppose that the $z = 3$ regime, where magnetic fields at super-horizon scales are generated, begins at $\eta = \eta_s$ and ends at $\eta = \eta_{\text{out}}$. Here, η_{out} is determined by $H(\eta_{\text{out}}) = M$ and we shall take the limit $\eta_s \rightarrow -\infty$ in the end of calculation. Then, the total energy density of magnetic fields for $\eta > \eta_{\text{out}}$ is

$$\epsilon_B(\eta) \simeq \int_{k_f(\eta_s)}^{k_f(\eta_{\text{out}})} \mathcal{P}_B \frac{dk}{k}, \quad (25)$$

where $k_f(\eta_{\text{out}})$ and $k_f(\eta_s)$ stand for the wavenumbers of fluctuations which freeze out at η_{out} and η_s , respectively. Here, for simplicity, we have assumed that $\kappa = O(1)$ or $|\kappa| \ll 1$ so that the $z = 2$ regime is short or absent. Then we see that $n = 2(5q-4)/(2q-1) > 0$ is necessary and sufficient for the finiteness of the integral in the limit $\eta_s \rightarrow -\infty$. Next, one wonders if electric fields affect the background universe. Since $\mathcal{P}_E \propto \mathcal{P}_B/k^2$, we realize that $n-2 > 0$ is necessary and sufficient for the finiteness of the total energy density of electric fields.

Together with the freeze-out condition of Eq. (13), we have the constraint for the power of the expansion rate as $q > 1$ ($1/2 < p < 1$).

At $\eta = \eta_{\text{out}}$, $\epsilon_B \sim M^4$ and the background energy density is around $H^2 M_{\text{pl}}^2 \sim M^2 M_{\text{pl}}^2$, where $M_{\text{pl}} \sim 10^{19} \text{ GeV}$ is the Planck scale. Then the backreaction from the generated magnetic field will be negligible if $M \ll M_{\text{pl}}$. Importantly, this is compatible with the lower bound (10).

In summary, the backreaction problem does not appear in the cases with $q > 1$ ($1/2 < p < 1$) and $M \ll M_{\text{pl}}$.

V. THE EVALUATION OF THE GENERATED MAGNETIC FIELD AT EQUAL TIME

In order to estimate the magnitude of generated magnetic fields, we need to specify the FRW background evolution in the early universe. For simplicity, we assume that an oscillating scalar field dominates the evolution of the background FRW universe in the early stage and then reheats the universe at $\eta = \eta_{\text{rh}}$. Thus, we set $q = 2$ for $\eta \leq \eta_{\text{rh}}$. We also suppose that $\eta_{\text{out}} < \eta_{\text{rh}} < \eta_{\text{eq}}$, where $\eta = \eta_{\text{eq}}$ corresponds to the matter-radiation equality. Then the scale factor behaves as

$$a = \begin{cases} \left(\frac{\eta_{\text{rh}}}{\eta_0}\right) \left(\frac{\eta_{\text{eq}}}{\eta_0}\right) \left(\frac{\eta}{\eta_{\text{rh}}}\right)^2 & \eta \leq \eta_{\text{rh}} \\ \left(\frac{\eta_{\text{eq}}}{\eta_0}\right)^2 \left(\frac{\eta}{\eta_{\text{eq}}}\right) & \eta_{\text{rh}} \leq \eta \leq \eta_{\text{eq}} \\ \left(\frac{\eta}{\eta_0}\right)^2 & \eta_{\text{eq}} \leq \eta \end{cases} \quad (26)$$

Let $\eta_{\text{cross}} (\eta_{\text{rh}} < \eta_{\text{cross}} < \eta_{\text{eq}})$ be the conformal time at which the fluctuation with the wavenumber k re-enters the horizon. Then the spectrum at matter-radiation equality ($\eta = \eta_{\text{eq}}$) is given by

$$\mathcal{P}_B(\eta_{\text{eq}}) = \left(\frac{a_{\text{cross}}}{a_{\text{eq}}}\right)^4 \mathcal{P}_B(\eta_{\text{cross}}), \quad (27)$$

where

$$\mathcal{P}_B(\eta_{\text{cross}}) = \left(\frac{\eta_{\text{cross}}}{\eta_{\text{rh}}}\right)^2 \left(\frac{a_{\text{rh}}}{a_{\text{cross}}}\right)^4 \mathcal{P}_B(\eta_{\text{rh}}). \quad (28)$$

Setting $z = 3$ and $q = 2$ (thus $n = 4$), $\mathcal{P}_B(\eta_{\text{rh}})$ can be computed from Eq. (24) as

$$\mathcal{P}_B(\eta_{\text{rh}}) = (\alpha M)^{2/3} k^4 \frac{\eta_{\text{rh}}^2}{a_{\text{rh}}^4}, \quad (29)$$

where $\alpha := \eta_{\text{eq}}/(\eta_{\text{rh}}\eta_0^2)$ so that $a = \alpha\eta^2$ for $\eta \leq \eta_{\text{rh}}$.

Since $\eta_{\text{out}} < \eta_{\text{rh}}$, the horizon re-entry occurs in the IR regime, where the sound horizon and the Hubble horizon agree. This implies that $\eta_{\text{cross}} \sim k^{-1}$. Thus, after some short calculations we obtain

$$\mathcal{P}_B(\eta_{\text{eq}}) \simeq k^2 M^{2/3} a_{\text{eq}}^{-7/2} H_{\text{rh}}^{1/3} \eta_0^{-1}, \quad (30)$$

where we have used the relations $\eta_{\text{eq}} = a_{\text{eq}}^{1/2} \eta_0$ and $\eta_{\text{rh}} \simeq H_{\text{rh}}^{-1/2} a_{\text{eq}}^{-1/4} \eta_0^{1/2}$.

Now we can evaluate the order of magnitude of the magnetic field at $\eta = \eta_{\text{eq}}$ as

$$\begin{aligned} B(\eta_{\text{eq}}) &\simeq \sqrt{\mathcal{P}_B(\eta_{\text{eq}})} \\ &\simeq 10^{-27} \left(\frac{k}{1\text{Mpc}^{-1}}\right) \left(\frac{M}{10^{-3}M_{\text{pl}}}\right)^{1/3} \left(\frac{a_{\text{eq}}}{10^{-3}}\right)^{-7/4} \\ &\quad \times \left(\frac{\eta_0}{13\text{Gyr}}\right)^{-1/2} \left(\frac{H_{\text{rh}}}{10^{13}\text{GeV}}\right)^{1/6} \text{Gauss}. \end{aligned} \quad (31)$$

At galactic scales, the primordial amplitude (31) can be amplified to the observed amplitude of galactic magnetic

fields by the dynamo mechanism, following the argument given in Ref. [25].

On the other hand, if there is no amplification mechanism then the equal-time amplitude (31) would correspond to about 10^{-33}Gauss or lower for scales of 1Mpc or longer at present time. This means that intercluster magnetic field predicted by our mechanism is too weak to be observed directly or indirectly [26]. As for cluster magnetic field, as stated in introduction, there is no consensus about how observed amplitudes could be related to primordial ones. It is, however, possible that the primordial magnetic fields generated by our mechanism at galactic scales could be amplified by the galactic dynamo and then spread over cluster scales [11, 12].

Finally we comment on the constraint from the Big Bang Nucleosynthesis (BBN). Since the abundance of the light elements is observed precisely, BBN gives the upper limit on the strength of the magnetic fields. The limits on the homogeneous magnetic fields on the BBN-horizon size ($\sim 10^{-4}\text{Mpc}$) are less than 10^{-6}Gauss in terms of today's values [28]. The magnetic field generated in our current model is $B \sim 10^{-29}\text{Gauss}$ on the BBN-horizon scale in terms of today's values. Thus we see that it is consistent with the BBN constraint.

VI. SUMMARY

We have presented a new mechanism for generation of large-scale magnetic field, based on the power-counting renormalizable theory of gravitation recently proposed by Hořava. Contrary to the usual case in general relativity, the $U(1)$ gauge symmetry of a Maxwell action in this theory permits terms breaking conformal invariance in the ultraviolet. Moreover, for high frequency modes, the anisotropic scaling intrinsic to the theory inevitably makes the sound horizon far outside the Hubble horizon. Consequently, non-inflationary cosmic expansion in the early universe naturally generates super-horizon quantum fluctuations of the magnetic field. Specializing our consideration to the case with the dynamical critical exponent $z = 3$, we have shown an explicit set of parameters for which (i) the amplitude of generated magnetic field is large enough as a seed for the dynamo mechanism; (ii) backreaction to the cosmic expansion is small enough; and (iii) the high-energy dispersion relation is consistent with the most recent observational limits from MAGIC and FERMI.

As stated in Sec. I, the value of $z (\geq 3)$ in the UV is a part of the definition of a theory, provided that the theory is renormalizable. In the present paper, we have restricted our consideration to the simplest case where z in the UV is 3. However, in principle the mechanism presented in this paper works for any values of z in the UV. For general z , radiation energy density scales as $\rho \propto a^{-(3+z)}$. This means that a radiation dominated epoch of the universe has a power-law expansion $a \propto t^p$ with $p = 2/(3+z)$. This expansion law satisfies the con-

dition (13), or $p > 1/z$, if $z > 3$. For this reason, if we consider a version of the Horava-Lifshitz theory with $z > 3$ then magnetic fields can be generated during a radiation dominated epoch. Further investigation of the mechanism with general z is certainly worthwhile.

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APPENDIX A: QUANTITATIVE ANALYSIS OF POWER SPECTRUM

Here we will carefully compute the power spectrum of the generated magnetic field. To do so we will take a gauge-fixing as usual (For example, see Ref. [27]). Without loss of generality, we can choose the gauge of $A_0 = 0$. Then we may want to take the transverse gauge of $\partial_i A_i = 0$. For example, we consider the UV action of $z = 3$. In this case one of the field equations is

$$\partial_t \partial_i A_i - \Delta A_0 = 0, \quad (\text{A1})$$

where $\Delta := \delta^{ij} \partial_i \partial_j$. From this we can easily see that $\partial_i A_i = 0$ holds for all t if we impose $\partial_i A_i = 0$ at an initial time. In addition, we can see that the gauge condition is consistent with the remaining field equations. Thus we adopt the gauge of $A_0 = \partial_i A_i = 0$ hereafter.

In the UV limit the action with the critical exponent z is approximately given by

$$S_{UV} = \frac{1}{2} \int d\eta \, d^3 \vec{x} \left[(\partial_\eta A_i)^2 + \frac{(-1)^{z+1}}{(aM)^{2z-2}} A_i \Delta^z A_i \right] \quad (\text{A2})$$

From now on we follow the conventional second quantization. First we expand the vector perturbation as

$$A_i = \int d^3 \vec{k} \sum_{\sigma=1,2} \left(b_{\vec{k},\sigma} u_{\vec{k}} \epsilon_i(\vec{k}, \sigma) + b_{\vec{k},\sigma}^\dagger u_{\vec{k}}^* \epsilon_i^*(\vec{k}, \sigma) \right) \quad (\text{A3})$$

where $\epsilon_i(\vec{k}, \sigma)$ ($\sigma = 1, 2$) is the orthonormal transverse polarization vector and the operators $b_{\vec{k}}$ and $b_{\vec{k}}^\dagger$ satisfy

the following commutators

$$\begin{aligned} [b_{\vec{k},\sigma}, b_{\vec{k}',\sigma'}^\dagger] &= (2\pi)^3 \delta_{\sigma\sigma'} \delta^{(3)}(\vec{k} - \vec{k}'), \\ [b_{\vec{k},\sigma}, b_{\vec{k}',\sigma'}] &= [b_{\vec{k},\sigma}^\dagger, b_{\vec{k}',\sigma'}^\dagger] = 0. \end{aligned} \quad (\text{A4})$$

The vacuum is defined by

$$b_{\vec{k},\sigma} |0\rangle = 0 \quad \text{for } \vec{k}. \quad (\text{A5})$$

The mode functions follows the Klein-Gordon normalization as usual:

$$\begin{aligned} (u_{\vec{k}}, u_{\vec{k}'}) &\equiv -i \int d^3 \vec{x} \left(u_{\vec{k}} \partial_\eta u_{\vec{k}'}^* - u_{\vec{k}'}^* \partial_\eta u_{\vec{k}} \right) \\ &= \frac{1}{(2\pi)^3} \delta^{(3)}(\vec{k} - \vec{k}'). \end{aligned} \quad (\text{A6})$$

Then the equation for the mode function becomes

$$u_{\vec{k}}'' + \frac{(-1)^z}{(aM)^{2z-2}} \Delta^z u_{\vec{k}} = 0, \quad (\text{A7})$$

where the prime stands for the derivative with respect to the conformal time. Introducing $\chi_{\vec{k}}$ as

$$u_{\vec{k}}(k, \eta) = \frac{e^{i\vec{k} \cdot \vec{x}}}{(2\pi)^3} \chi_{\vec{k}}(\vec{k}, \eta), \quad (\text{A8})$$

the equation for $\chi_{\vec{k}}$ becomes

$$\chi_{\vec{k}}'' + \frac{k^{2z}}{(aM)^{2z-2}} \chi_{\vec{k}} = 0. \quad (\text{A9})$$

Here we define that the physical frequency is $\omega := k^z / (a^z M^{z-1})$.

Hereafter, we assume that the scale factor has the power law, $a = \alpha \eta^q$, with $q = p/(1-p) > 1/(z-1)$ which comes from the condition of Eq. (13). Then the equation for $\chi_{\vec{k}}$ becomes

$$\chi_{\vec{k}}'' + \beta_k \eta^{-(2z-2)q} \chi_{\vec{k}} = 0, \quad (\text{A10})$$

where $\beta_k := k^{2z} / (\alpha M)^{2z-2}$. The solution is given by

$$\begin{aligned} \chi_{\vec{k}} &= C_1 \sqrt{\eta} H_\nu^{(1)} \left(-2\nu \sqrt{\beta_k} \eta^{1/(2\nu)} \right) \\ &\quad + C_2 \sqrt{\eta} H_\nu^{(2)} \left(-2\nu \sqrt{\beta_k} \eta^{1/(2\nu)} \right), \end{aligned} \quad (\text{A11})$$

where $\nu := -1/2(qz - q - 1)$ and $H_\nu^{(n)}$ is the ν th-order Hankel functions. From the normalization and choosing the mode function to be the positive-frequency mode in Minkowski spacetime at short-wavelength limit, C_1 and C_2 are fixed and then

$$u(\vec{k}, \eta) = \frac{e^{i\vec{k} \cdot \vec{x}}}{(2\pi)^3} \sqrt{-\frac{\pi \nu \eta}{2}} H_\nu^{(1)} \left(-2\nu \sqrt{\beta_k} \eta^{1/(2\nu)} \right) e^{i\pi \frac{2\nu+1}{4}}. \quad (\text{A12})$$

Now we can calculate the power spectrum of A_i which is defined as

$$\langle 0 | A_{i,\vec{k}} A_{j,\vec{k}'} | 0 \rangle \equiv (2\pi)^3 \delta_{ij} \delta^{(3)}(\vec{k} + \vec{k}') \frac{2\pi^2}{k^3} \mathcal{P}_{A_i}, \quad (\text{A13})$$

where

$$A_{i,\vec{k}} = \int d^3\vec{x} e^{i\vec{k}\cdot\vec{x}} A_i(\vec{x}). \quad (\text{A14})$$

Then the power spectrum for A_i will be

$$\begin{aligned} \mathcal{P}_{A_i} &= \frac{k^3}{2\pi^2} |(2\pi)^3 u_{\vec{k}}|^2 \\ &= \frac{k^3}{2\pi^2} \left(-\frac{\pi\nu\eta}{2} \right) \left| H_{\nu}^{(1)} \left(-2\nu\sqrt{\beta_k}\eta^{1/(2\nu)} \right) \right|^2 \end{aligned} \quad (\text{A15})$$

Therefore, the power spectrum of the magnetic field is

$$\mathcal{P}_B \simeq \frac{k^2}{a^4} \mathcal{P}_{A_i}. \quad (\text{A16})$$

Now we estimate \mathcal{P}_{A_i} and \mathcal{P}_B in $\eta \rightarrow \infty$ when the fluctuations freeze out ($H \gg \omega$). The mode function will

be approximated by

$$u(\vec{k}, \eta) = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^3} \sqrt{\frac{-\nu}{2\pi}} \eta \Gamma(-\nu) \left(-\nu\sqrt{\beta_k} \right)^\nu e^{i\pi\frac{2\nu+1}{4}}. \quad (\text{A17})$$

Then the power spectrum is obtained as

$$\mathcal{P}_{A_i}|_{H \gg \omega} = \frac{k^3(-\nu)^{2\nu+1}}{4\pi^3} (\eta \Gamma(-\nu))^2 (\beta_k)^\nu. \quad (\text{A18})$$

Finally $\mathcal{P}_B|_{H \gg \omega}$ becomes

$$\begin{aligned} \mathcal{P}_B|_{H \gg \omega} &\simeq \frac{k^5(-\nu)^{2\nu+1}}{4\pi^3 a^4} (\eta \Gamma(-\nu))^2 (\beta_k)^\nu \\ &\simeq (\alpha M)^{(z-1)/(qz-q-1)} k^n \frac{\eta^2}{a^4}, \end{aligned} \quad (\text{A19})$$

where $n = 5 - z/(qz - q - 1)$. This result agrees with that of Eq. (24) using the scaling law argument in the text.

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